# Is the Spectrum of Gravitational Waves the "Holy Grail" of Inflation?

Robert H. Brandenberger*

*Department of Physics, McGill University, Montréal, QC, H3A 2T8, Canada*

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It is often said that detecting a spectrum of primordial gravitational waves via observing B-mode polarization of the Cosmic Microwave Background is the "Holy Grail" of inflation. The purpose of this short note is to point out that it is indeed of immense scientific interest to search for a signal of gravitational waves in B-mode polarization. However, rather than proving that inflation is the right paradigm of early universe cosmology, a positive signal of direct primordial B-mode polarization might well be due to other sources than inflation. In fact, a careful characterization of the spectrum of B-mode polarization might even falsify the inflationary paradigm.

## I. OVERVIEW

At the present time, the Cosmic Microwave Background (CMB) is our most accurate tool to study the structure of the universe. CMB temperature maps allow us to study the structure of the universe at the time $t_{rec}$ when the microwave photons last scattered. The current high precision maps provide strong support for the paradigm that the structure which we now see in our universe comes from a primordial spectrum of almost adiabatic and almost scale-invariant fluctuations which were present on super-Hubble scales already before $t_{rec}$. The detailed quantitative analysis of these maps also allows us to tightly constrain a number of cosmological parameters which describe our background cosmology. The CMB temperature maps are now so accurate that they are cosmic variance limited for all larger angular scales.

There is, however, more information in the CMB than simple temperature maps reveal: CMB radiation is polarized, and the polarization carries a lot of important information about cosmology. CMB polarization can be decomposed into E-mode and B-mode polarization. The E-mode polarization power spectrum has now been observed, as has the cross correlation power spectrum between temperature and E-mode. However, to date no B-mode polarization has been detected.

Primordial adiabatic density fluctuations only lead to E-mode polarization, whereas gravity waves will lead to a signal in B-mode polarization. Thus, the search for B-mode CMB polarization is one of the ways to detect a spectrum of primordial gravitational radiation on cosmological scales.

The inflationary scenario [1, 2] is the current paradigm for early universe cosmology. Inflation solves some of the mysteries of Big Bang Cosmology and is the first model based on principles of relativistic physics to predict [3] (see also [4, 5]) an approximately scale-invariant spectrum of approximately adiabatic cosmological fluctuations [55]. Inflationary models also predict a roughly

scale-invariant spectrum of graviational waves [4].

It is currently often said that the measurement of gravitational radiation through the detection of primordial B-mode polarization is the "Holy Grail" needed to confirm that inflation is the correct theory of the very early universe (see e.g. [6]). The purpose of this note is to put this claim into some perspective. Implicit in the claim that detecting primordial B-mode polarization will be the "Holy Grail" of inflation are two assumptions: firstly that there are no other sources of direct B-mode polarization except for gravitational waves, and secondly that if indeed gravitational waves are responsible for B-mode polarization, then these waves will come from inflation.

Here, I point out that IF there is a measurable spectrum of gravitational waves on cosmological scales, these waves could well be due to other sources than inflation. Whereas inflation typically produces a small amplitude of gravitational waves, the other sources which I will mention can produce a larger amplitude. Secondly, I point out that there are other cosmological sources of B-mode polarization. Hence, a positive detection of B-mode polarization might NOT be due to gravitational waves at all.

This Note is intended mainly for experimentalists. I hope to argue here that it is even more important to search for B-mode polarization than if gravitational waves from inflation were the only possible source. In fact, I will show that it is possible that B-mode polarization results could falsify the inflationary paradigm.

## II. WHY LOOK BEYOND INFLATIONARY COSMOLOGY?

The scenario of cosmological inflation [1] has for good reasons become the paradigm of early universe cosmology. It explains mysteries about the structure of the universe which Standard Cosmology could not address such as the horizon, flatness and entropy problems. More importantly, it provides a theory for the origin of an almost scale-invariant spectrum of primordial adiabatic curvature fluctuations on scales which at $t_{rec}$ are larger than the Hubble radius (the apparent horizon). Such a spec-
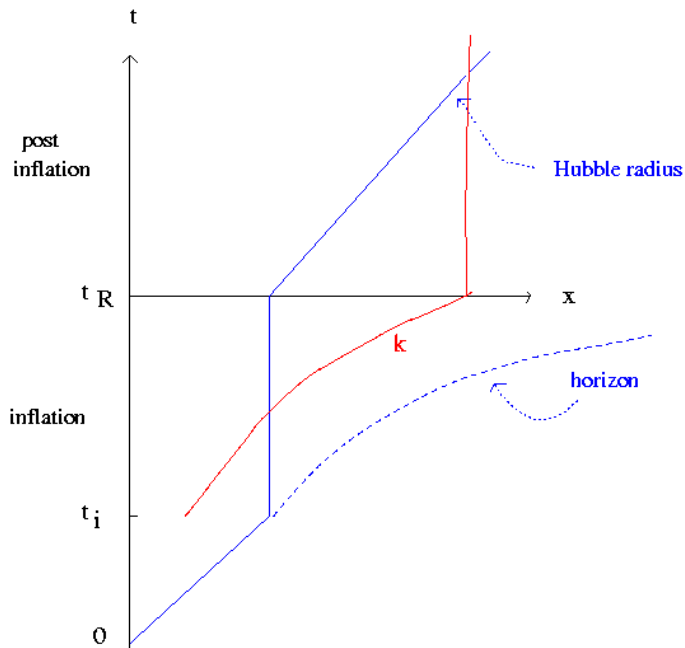
FIG. 1: Space-time sketch of inflationary cosmology. The vertical axis is time, the horizontal axis corresponds to physical distance. The period of inflation lasts from $t_i$ until $t_R$. Before $t_i$ and after $t_R$ the universe is in a phase dominated by radiation. The time $t_R$ can be viewed as the start of the the Standard Cosmology epoch of the inflationary scenario. The solid line labelled $k$ is the physical distance between two points at rest in the expanding space. This distance corresponds to the scale of a fluctuation. Note that the horizon (the forward light cone of a point at the Big Bang) begins to deviate exponentially from the Hubble radius as the inflationary phase proceeds.

trum is in excellent agreement with the current data.

A space-time sketch of inflationary cosmology is shown in Figure 1. The vertical axis is time, the horizontal axis represents physical spatial distance. The inflationary phase lasts from time $t_i$ until the time $t_R$ of "reheating". During this phase, space is expanding almost exponentially and hence the Hubble radius (defined as the inverse expansion rate) is approximately constant [56] During the inflationary phase, the physical wavelength of a fluctuation mode increases exponentially. It is this expansion which leads to the possibility that the fluctuation modes on large cosmological scales which are currently being probed by observation emerge at the beginning of the inflationary period on sub-Hubble scales, which allows a causal theory for their origin. In inflationary cosmology it is assumed that the fluctuations emerge at some early time as quantum vacuum perturbations on sub-Hubble scales [3] (see also [4, 5]). The success of inflation as a theory that can explain the origin of structure in the universe is based on two key points: Firstly, scales which we observe today need to have a wavelength smaller than the Hubble radius at some initial time. Secondly, the fluctuations need to propagate freely for a long dura-

tion of time on super-Hubble scales. This is necessary to establish the coherence of the phases of different fluctuation modes which in turn is responsible for producing the acoustic oscillations in the angular power spectrum of the CMB. Any alternative to inflation will have to contain these two features.

It is important to realize that a scale-invariant spectrum of adiabatic fluctuations as the origin of structure in the universe was already postulated a decade before the development of inflationary cosmology [7], and that ANY scenario which produces an almost scale-invariant spectrum of adiabatic fluctuations agrees equally well with the observations. This point is generally recognized, but it is then often claimed that since inflation produces a nearly scale-invariant spectrum of gravitational waves, the discovery of such a spectrum will confirm inflation. However, as will be emphasized here, this logic is incorrect.

Part of the motivation for looking beyond inflationary cosmology is the fact that (at least current realizations of) inflation suffer from various conceptual problems (see e.g. [10] for more extended discussions of these problems). Current models of inflation are based on coupling scalar matter fields to Einstein gravity. Whereas it is easy to construct toy models of inflation, it has proven very difficult - in spite of thirty years of efforts - to embed inflation into a fundamental theory of matter (see e.g. [11] for reviews of attempts to embed inflation into superstring theory). Even if a candidate model with a scalar field which can lead to inflation is found, then often special initial conditions need to be chosen in order to obtain inflation, and the parameters in the model need to be finely tuned if the observed amplitude of the spectrum of fluctuations is to emerge (this is the so-called "amplitude problem" of inflationary cosmology).

There are, however, more serious problems. One of them is the "trans-Planckian problem" for fluctuations [12, 13]: if the period of inflation lasts only slightly longer than it needs to in order for inflation to be successful at solving the cosmological problems of the Standard Big Bang which it is designed to solve [57], then the wavelengths of all scales which are currently probed in cosmological observations are smaller than the Planck length at the beginning of the inflationary period. The physical laws which underlie inflationary cosmology break down on length scales smaller than the Planck length (possibly much earlier), and thus the usual computations of the spectrum of cosmological fluctuations have been done using a theory which does not apply in the zone of ignorance indicated in Figure 2. It is easy to construct [13] toy models of physics beyond the Planck scale which yield large deviations from the usual predictions of inflation.

If inflation is obtained by coupling classical matter fields to Einstein gravity, then a cosmological singularity in the past is as unavoidable [14] as it is in Standard Cosmology. Thus, in the same way that we knew that Standard Cosmology could not be the ultimate theory of the very early universe, we know that neither can in-
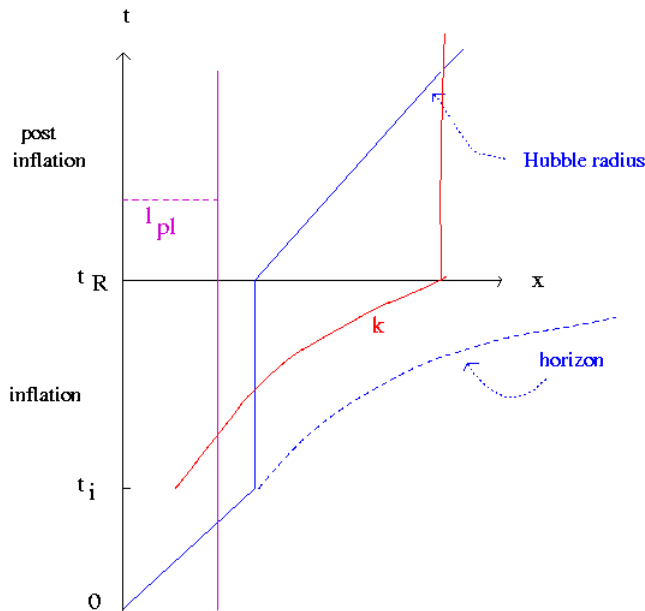
FIG. 2: A space-time sketch of inflationary cosmology. As in Figure 1, the vertical axis is time, the horizontal axis denotes physical distance. In addition to the Hubble radius, the horizon and the physical wavelength corresponding to a fluctuation mode which expands as the size of space increases (the curve denoted by $k$), we show the vertical curve coresponding to the Planck length. Unless the period of inflation is only very slightly larger than the minimal duration it has to have to solve the problems of Standard Cosmology, the physical wavelength corresponding to our current Hubble radius is smaller than the Planck length at the beginning of inflation. Thus, all scales which are observable now in cosmology emerge from the short wavelength zone of ignorance. This is an illustration of the trans-Planckian problem.
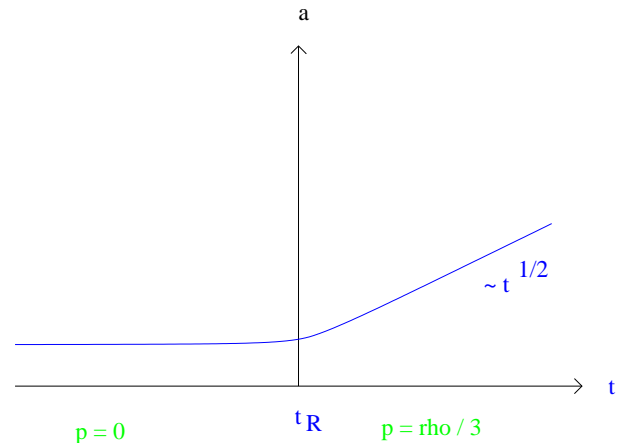


FIG. 3: Time evolution of the scale factor (size of space) in the emergent universe scenario. The time axis is the horizontal axis, the vertical axis gives the value of the scale factor. The time $t_R$ is the end of the emergent phase, the time when the universe makes the transition to the radiation stage of Standard Cosmology during which the scale factor increases as the square root of time.

flationary cosmology. There must be a theory beyond inflation which applies at very high densities. It may be the case that this theory contains inflation as the low density limit, but this need not be the case.

In large field toy models of inflation, the energy scale at which the inflationary phase takes place is typically of the order $10^{16}$GeV, much closer to the Planck scale than to scales where the theory has been tested. In all approaches to quantum gravity it emerges that the Einstein action does not yield a good approximation to the dynamics at scales that approach the Planck scale. Since the scale of inflation is so close to the Planck scale (and even closer to the string scale [15]), it may be dangerous to trust the conclusions obtained by applying Einstein gravity.

The "Achilles heel" of inflation is related to the cosmological constant problem which arises whenever we couple quantum matter to gravity. Quantum vacuum energy cannot gravitate since if it would it would lead to a cosmological constant which is at least sixty orders of magnitude larger than the observed one. Thus, there must be a mechanism which renders quantum vacuum energy gravitationally inert. Why doesn't the same mechanism

render the constant part of a scalar field potential energy (which leads to inflation) gravitationally inert?

The above conceptual problems of inflation motivate the search for alternatives. As will be shown below, there are in fact alternative cosmological scenarios, and some of them yield an almost scale-invariant spectrum of gravitational waves like inflationary cosmology does. In fact, in at least one of these alternative approaches, the amplitude of the gravitational wave spectrum is larger than it is in simple inflationary models.

## III. ALTERNATIVES TO INFLATION

In this section I will briefly review three alternative scenarios to inflation. This list is not meant to be complete! The three scenarios I will mention are an "emergent stringy universe" [16, 17], the "matter bounce scenario" (see e.g. [18] for a review of this and other alternatives), and the Ekpyrotic universe [19]. Two of the three produce a spectrum of almost scale-invariant gravitational waves on cosmological scales, both with amplitudes comparable or larger than what small field inflationary models produce.

### A. Emergent Stringy Universe

In the "emergent universe" scenario [20] it is postulated that the universe begins in a long quasi-static phase which at some point in time $t_R$ makes a transition to the expanding universe of Standard Big Bang Cosmology. The evolution of the scale factor as a function of time is sketched in Figure 3, the horizontal axis being time,
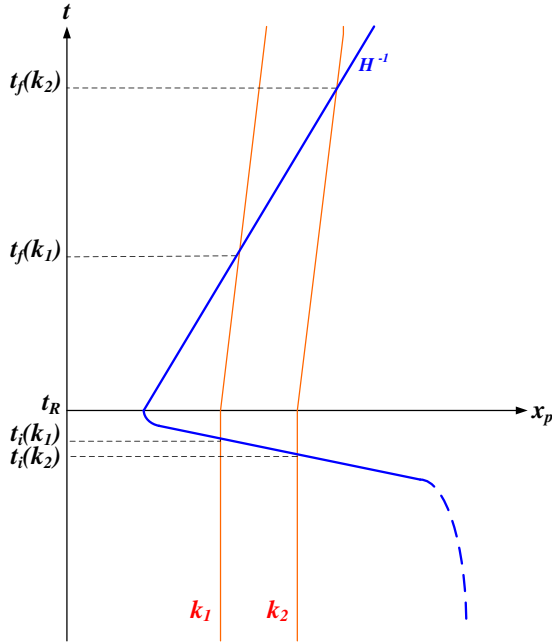
FIG. 4: Space-time sketch of the emergent universe scenario. As in Figure 1 in the illustration of inflationary cosmology, the vertical axis indicates time, the horizontal axis physical distance. The time $t_R$ is the end of the emergent phase and the onset of the radiation phase and plays a similar role to the time of reheating $t_R$ in inflationary cosmology. The Hubble radius (the blue solid curve labelled by $H^{-1}$) is infinite deep in the emergent phase. The physical wavelengths of fluctuation modes (indicated by $k_1$ and $k_2$ in the graph) are constant in the emergent phase. A mode exits the Hubble radius towards the end of the emergent phase at the time $t_i(k)$ and re-enters in late time cosmology at the time $t_f(k)$.

the vertical axis indicating the value of the scale factor. A sketch of the resulting space-time is given in Figure 4. As in Figures 1 and 2, the vertical axis is time and the horizontal axis indicates physical distance. We plot the Hubble radius and the wavelength of some fluctuation mode. Since the universe is quasi-static before $t_R$, the Hubble radius tends to infinity. Hence, fluctuations which we observe today originate on sub-Hubble scales during the early "emergent" phase. After $t_R$, the fluctuations propagate on super-Hubble scales until they re-enter the Hubble radius at late times. Thus, the two key properties allowing for the existence of a causal mechanism to produce primordial cosmological fluctuations and which also lead to acoustic oscillations in the CMB angular power spectrum (see the discussion at the end of the second paragraph of Section 2) are satisfied.
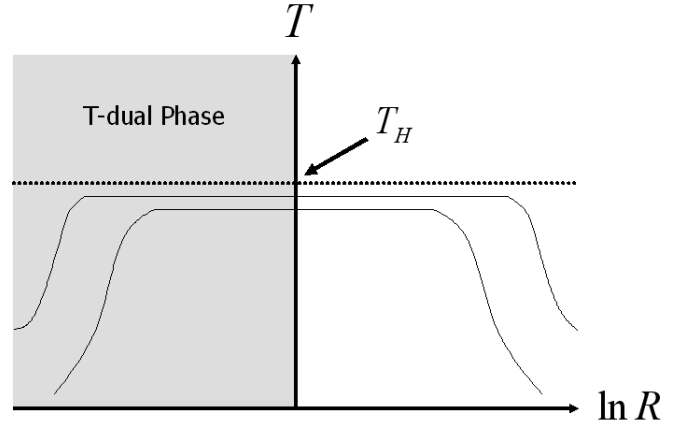


FIG. 5: Evolution of the temperature (vertical axis) as a function of the logarithm of the radius (horizontal axis). Two possible evolutionary tracks are shown which differ in the amount of entropy which they contain. If we begin the evolution at large radii, then initially the temperature rises as space contracts as for point particle matter. However, once the temperature approaches the limiting Hagedorn temperature $T_H$, the temperature ceases to rise as $R$ decreases. Below a critical radius which is chosen to be one in the figure, the temperature decreases as $R$ decreases.

A possible realization of the emergent universe scenario arises in the context of "String Gas Cosmology" [16] (see [21] for reviews and [22] for some other original work). If we couple a gas of closed fundamental strings to a space-time background in the same way that in Standard Cosmology we couple a gas of point particles to a background space-time, then very important differences to point particle cosmology emerge at high densities. In particular, there is a maximal temperature for a gas of strings in thermal equilibrium, the so-called Hagedorn temperature $T_H$ [23]. If we compress a box of strings, then we observe that initially the temperature $T$ will rise, until $T$ approaches $T_H$. In the case of a compact space and a gas of closed strings, the evolution of $T$ as a function of the box radius $R$ is sketched in Figure 5 [58]. In the string gas cosmology scenario [16] it is postulated that the universe begins in a quasi-static phase where the temperature hovers just below $T_H$. Eventually, the winding string modes which are important in the quasi-static phase decay into string loops, and this leads to the beginning of the radiation phase of Standard Cosmology.

In string gas cosmology, matter in the early universe is a gas of strings in thermal equilibrium. Hence, the thermal fluctuations dominate over the vacuum ones - in contrast to inflationary cosmology where it is assumed that fluctuations arise from a vacuum state. It was realized in [17] that in a compact space, thermal fluctuations of strings in the early phase of string gas cosmology lead to a nearly scale-invariant spectrum of cosmological perturbations. The amplitude of the spectrum depends on the ratio of the string scale to the Planck scale, and if that number is taken to be the preferred value in the

string theory textbook [15], then the resulting amplitude of the perturbation spectrum is in good agreement with the data. Thus, string gas cosmology can successfully address some of the conceptual problems of the inflationary scenario: there is no amplitude problem for the magnitude of the spectrum, and since the wavelengths of fluctuation modes never become smaller than the length they have at $t_R$ (which is of the order of 1mm for the scale corresponding to the current Hubble radius, and hence many orders of magnitude larger than the Planck scale), modes we observe today are never close to the short wavelength zone of ignorance. Hence the trans-Planckian problem for fluctuations is absent.

### B. Matter Bounce Scenario

A second alternative to inflation is the "matter bounce" scenario. If there were a non-singular bouncing cosmology, then there would obviously be no singularity problem. Penrose and Hawking a long time ago taught us that in order to obtain such a non-singular bounce, one needs to go beyond General Relativity as the theory of space-time, or else invoke matter which violates some of the usual energy conditions. There is a long history of such attempts (see [24] for a comprehensive list of references). For a recent construction in the context of a modified gravitational theory (namely Hořava-Lifshitz gravity [25]) see [26] and for another recent construction using ghost condensate matter see [27].

Given a non-singular cosmology, it is most logical to assume that the contracting phase begins with a matter-dominated phase, then makes a transition to a radiation-dominated phase before undergoing the bounce and re-emerging as an expanding Standard cosmology. Figure 6 presents a space-time sketch of a bouncing cosmology. As in earlier figures, the vertical axis represents time, the horizontal axis physical distance. The Hubble radius and the physical wavelength of perturbation modes are symmetric about the bounce point. Thus, we see immediately that fluctuations originate at early times on sub-Hubble scales, and that they propagate for a long time on super-Hubble scales. Thus, the two key conditions for being able to generate primordial curvature fluctuations which agree with the observed acoustic oscillations in the angular power spectrum of the CMB are satisfied [59] . Based on the second law of thermodynamics we know that the duration of the radiation phase of contraction cannot be longer than that of the expanding radiation phase. Hence, scales which are currently observed have exited the Hubble radius in the matter-dominated phase of contraction.

A while back it was realized [28, 29] that initial vacuum fluctuations on sub-Hubble scales in the contracting phase evolve into a scale-invariant spectrum of curvature perturbations on super-Hubble scales before the bounce. As long as the bounce phase does not change this spectrum (which it does not in a number of toy models for
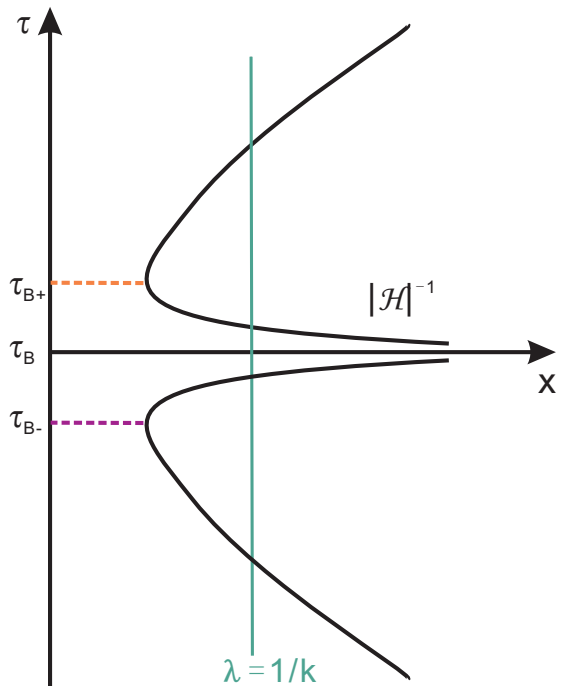
FIG. 6: Space-time sketch of the matter bounce scenario. The vertical axis is conformal time $\eta$, the horizontal axis denotes a comoving space coordinate. Also, $\mathcal{H}^{-1}$ denotes the co-moving Hubble radius. The vertical solid green line indicates the wavelength of a particular perturbation mode. As is evident, this mode exits the Hubble radius at some point during the contracting phase and re-enters at a later time in the expanding Standard Big Bang phase.

bouncing cosmologies in which the evolution of the fluctuations can be studied (see e.g. [30]), we get a scale-invariant spectrum at late times. This is called the "matter bounce" alternative to cosmological inflation as the explanation for the origin of structure.

### C. The Ekpyrotic Universe

The major problem of the "matter bounce" scenario and of other bouncing cosmologies is the "anisotropy problem": during the phase of contraction, the energy density in anisotropies grows faster than the energy density in the isotropic matter components. Hence, unless the initial anisotropies are tuned to be extremely small, no smooth cosmological bounce can occur.

The Ekpyrotic scenario [19] is a bouncing cosmology which avoids this problem. The model is motivated by a higher dimensional string theoretical framework [31] in which our matter is confined to a four space-time dimensional "brane" which is one of the two boundaries of a five dimensional bulk space-time. The extra spatial dimension is an interval of finite length, bounded on each side by a brane. There is an attractive force between the two boundary branes which renders the separation $r$ between the branes dynamical. From the point of view of our four

dimensional physics on the brane, the separation of the branes corresponds to a scalar field $\varphi$ with $\varphi \sim \log r$.

The Ekpyrotic scenario assumes that the attractive potential $V(\varphi)$ is a negative exponential function, leading to slow contraction of space. Once the separation between the branes becomes comparable to the string scale, quantum effects take over, leading (as is postulated but not proven) to a cosmological bounce. The radiation generated during the bounce leads to a radiation dominated phase of expansion. In contrast to the "matter bounce" scenario, the time evolution is not symmetric about the bounce point! Due to their coupling with $\varphi$, the scalar cosmological fluctuations acquire a scale-invariant spectrum in the contracting phase which, when taking extra-dimensional effects or entropy fluctuations into account (see e.g. [32]) leads to a scale invariant spectrum of curvature fluctuations after the bounce.

## IV. GRAVITATIONAL WAVES FROM INFLATION

In inflationary cosmology, a scale-invariant spectrum of gravitational waves is generated from initial quantum vacuum fluctuations of the canonical fields which represent each polarization state. We begin with the metric

$$ds^2 = a^2(\eta)\big[d\eta^2 - (\delta_{ij} + h_{ij})dx^i dx^j\big], \qquad (1)$$

where $h_{ij}$ is a transverse and traceless tensor which describes gravitational waves about a cosmological background given by the scale factor $a(\eta)$. We are using conformal time $\eta$ which is related to the physical time $t$ via $dt = a(\eta)d\eta$. The tensor $h_{ij}$ can be decomposed into the contributions from the two polarization states:

$$h_{ij}(\eta, x) = h_+(\eta, x)e_{ij}^+ + h_-(\eta, x)e_{ij}^-, \qquad (2)$$

where $e_{ij}^+$ and $e_{ij}^-$ are the two fixed polarization tensors, and $h^+$ and $h^-$ are the two coefficient functions which are scalar fields on the space-time background.

Inserting the gravitational wave ansatz (1) into the Einstein action and expanding to quadratic order in $h^+$ and $h^-$, it is straightforward to show that the resulting action for the fluctuations takes canonical form when written in terms of the fields

$$u^a \equiv ah^a, \qquad (3)$$

where the value of the index $a$ is either $+$ or $-$. The linear equations of motion for the Fourier modes $u_k$ of $u$ hence take on the simple form

$$u_k'' + \big(k^2 - \frac{a''}{a}\big)u_k = 0, \qquad (4)$$

where a prime indicates the derivative with respect to $\eta$. This equation shows that $u_k$ oscillates on scales smaller than the Hubble radius but is squeezed on super-Hubble scales [33].

Up to this point, the analysis has been completely general. Let us now consider the application to inflationary cosmology. In this case, the perturbations originate as quantum vacuum fluctuations. Since the amplitude of the quantum vacuum perturbations is given by the Hubble constant $H$, the power spectrum of gravitational waves at late times is

$$\mathcal{P}_h(k) \sim \frac{H(k)}{m_{pl}}, \qquad (5)$$

where $H(k)$ is the value of the Hubble constant during the inflationary phase when the scale $k$ exits the Hubble radius, and $m_{pl}$ is the Planck mass.

Note that if inflation is realized in the context of Einstein gravity and is generated by matter fields obeying the "Null Energy Condition", then the Hubble constant must be a decreasing function of time and hence $H(k)$ decreases as $k$ increases. Inflationary cosmology thus produces a roughly scale-invariant spectrum of gravitational waves with a tilt which has to be red. This result is to be contrasted with the tilt of the spectrum of cosmological perturbations produced by inflation which can be blue (although more often it is red, too).

The rate at which the gravitational waves are squeezed on super-Hubble scales is the same as the rate at which a test scalar field on a fixed background metric is squeezed. If the equation of state of the background is independent of time, the scalar metric fluctuations (the "cosmological perturbations") are squeezed at the same rate (this is true e.g. in the matter bounce scenario during the relevant time intervals), but it is NOT true in inflationary cosmology where the cosmological perturbations are amplified more during the reheating period than the gravitational waves, which leads to the small value of the tensor to scalar ratio $r$ which inflationary models generally predict:

$$r(k) \equiv \frac{\mathcal{P}_h(k)}{\mathcal{P}_\zeta(k)} \ll 1, \qquad (6)$$

where $\mathcal{P}_\zeta$ is the power spectrum of curvature fluctuations. Note that large-field inflation models produce a value of $r$ which is closer to 1 than small field models.

## V. ALTERNATIVE SOURCES OF GRAVITATIONAL WAVES

### A. Gravitational Waves from Topological Defects in Standard Cosmology

Before discussing what kinds of spectra of gravitational waves the alternative cosmological scenarios mentioned in Section 3 lead to, it is important to point out that even in Standard Cosmology there are processes that lead to a scale-invariant spectrum of gravitational waves. Specifically, I am referring to gravitational waves produced by a scaling distribution of topological defects such as cosmic strings [34].

Cosmic strings are linear topological defects predicted to form in a wide range of phase transitions of matter in the early universe. A subset of grand unified field theories leads to cosmic strings, in particular in many supergravity models [35]. Cosmic strings also form at the end of inflation in a wide set of inflationary models which arise motivated by superstring theory [36]. Cosmic strings are characterized by a mass per unit length $\mu$ (which is conventionally expressed in terms of the dimensionless number $G\mu$, $G$ being Newton's gravitational constant). The current upper bound on $\mu$ is about $G\mu < 2 \times 10^{-7}$ from the precision data on the angular power spectrum of CMB fluctuations in the region of the first acoustic peak [37].

In models which admit topologically stable cosmic strings, a network of such strings will inevitably form in the early universe (see [34] for detailed discussions and references to the original literature). It can be argued analytically and confirmed numerically that the string distribution will rapidly approach a "scaling solution" characterized by a network of infinite strings with a typical curvature radius which at all times $t$ is of the order the Hubble radius $t$, and a distribution of string loops with radii $R < \alpha t$ which looks the same at all times if distances are scaled to the Hubble radius. In the above, $\alpha$ is a number smaller than 1 which needs to be determined by numerical simulations. The loops are formed via intersections of segments of the infinite string network. They oscillate and gradually decay via gravitational radiation.

Due to the fact that the distribution of strings is scaling, a scale-invariant spectrum of gravitational waves emerges. In turn, this produces a tensor contribution to the microwave anisotropies, as first calculated in [38] (see also [39] for other studies of gravitational radiation from a network of cosmic strings).

Based on the scaling solution of the network of infinite strings, one can calculate the distribution of cosmic string loop sizes. The number density $n(R,t)$ in space of loops per $R$ interval at time $t$ (i.e. $n(R,t)dR$ is the number density in space of loops with radii in the interval $[R, R+dR]$), then for times $t \gg t_{eq}$ (where $T_{eq}$ is the time of equal matter and radiation) the distribution is given by [40]:

$$
\begin{aligned}
n(R,t) &= \nu R^{-2} t^{-2} \quad R \gg t_{eq} \\
n(R,t) &= \nu R^{-5/2} t_{eq}^{1/2} t^{-2} \quad \gamma G\mu t < R \ll t_{eq} .
\end{aligned} \quad (7)
$$

For smaller values of $R$, $n(R,t)$ is constant. The constant $\gamma$ determines the overall strength of gravitational radiation, and $\nu$ is a coefficient which depends on the number per Hubble volume of infinite string segments in the scaling solution.

Based on this scaling distribution of string loops, the energy density distribution in gravitational waves can be calculated, and the resulting CMB temperature anisotropies can be inferred [38]. The CMB anisotropy spectrum is scale invariant on large angular scales and has a r.m.s. amplitude which is given on angular scales

$\chi$ larger than about two degrees by [38]

$$
\begin{aligned}
< \big(\frac{\delta T}{T}\big)^2(\chi) >^{1/2} &\sim [\frac{144}{5}\pi\beta\gamma\nu]^{1/2} \\
&\sim 3 \times 10^{-6}\big(\frac{G\mu}{2 \times 10^{-7}}\big) \quad (8)
\end{aligned}
$$

where in the final step we have inserted typical values $\beta = 9$, $\nu = 10^{-2}$ and $\gamma = 5$ which follow from cosmic string network evolution simulations [41] [60]. In the above, $\beta$ is a constant of the order $2\pi$ which gives the length of a typical string loop as a multiple of the mean loop radius.

The above result for the amplitude of the tensor mode should be compared with the observed amplitude of the scalar mode which is

$$
< \big(\frac{\delta T}{T}\big)^2(\chi) >^{1/2} \sim 10^{-5}, \quad (9)
$$

on large angular scales. Thus, the predicted tensor to scalar ratio $r$ is

$$
r \sim 3 \times 10^{-1}\big(\frac{G\mu}{2 \times 10^{-7}}\big). \quad (10)
$$

For strings with tension close to the current observational bound this is a value of $r$ greater than what most inflationary models predict.

Although cosmic strings predict a spectrum of gravitational waves which is scale-invariant on large angular scales, the model predicts specific non-Gaussian signatures in position space. In CMB temperature maps the signal is a distribution of edges across which the temperature jumps [43]. These edges are due to strings present between the time $t_{rec}$ of last scattering and the present time $t_0$. String segments created at time $t$ lead to discontinuity lines in CMB maps of comoving scale $s_c$ given by

$$
s_c \sim t\big(z(t) + 1\big), \quad (11)
$$

where $z(t)$ is the cosmological redshift at time $t$. These edges can be searched for using edge detection algorithms (see e.g. [44–46] for recent studies).

As pointed out in [47], wakes behind moving strings present between the time $t_{eq}$ of equal matter radiation and the present time lead to a specific polarization signal. Most importantly, the extra scattering which photons passing though a wake at time $\tilde{t}$ experience leads to rectangular regions in the sky (with comoving scale given by (11)) of extra polarization. When averaged over all strings, there is an equal distribution of E-mode and B-mode polarization. This example shows that the discovery of cosmological B-mode polarization is not necessarily due to primordial gravitational waves.

Since the polarization signal of cosmic strings also has special non-Gaussian features, it is important to search for them in position space through the use of statistics such as the edge detection algorithms mentioned above.

As initially pointed out in [48], phase transitions in Standard Cosmology can lead to a scale-invariant spectrum of gravitational waves, even if they do not produce

defects. This mechanism is further discussed in [49]. As pointed out in these works, a way to potentially differentiate between inflation and phase transitions as the source of gravitational waves is the absence of characteristic acoustic oscillations in the angular power spectrum in the second case.

After this discussion of gravitational waves and direct B-mode polarization produced by a source which might be present in Standard Cosmology, let us now turn to the predictions for the spectrum of the stochastic background of gravitational radiation from the alternative cosmological models introduced earlier.

### B. Gravitational Waves from String Gas Cosmology

We have seen in Section 3 that String Gas Cosmology leads to an almost scale-invariant spectrum of cosmological perturbations. What is important for this Note is that string gas cosmology also leads to an almost scale-invariant spectrum of primordial gravitational waves [50]. The slope of the spectrum is predicted to be slightly blue, and not red as inflationary universe models predict [61] The amplitude of the gravitational wave spectrum and the resulting tensor to scalar ratio $r$ can also be calculated from first principles [50]. The value of $r$ is related by a consistency relation to the magnitude of the (red) tilt $n_s$ of the spectrum of cosmological perturbations:

$$r \sim |n_s - 1| . \tag{12}$$

Comparing this result to what is obtained in inflationary models, we see that in both scenarios, the typical value of $r$ is substantially smaller than $r = 1$. Which scenario gives rise to a larger amplitude depends very much on the special realization of the scenario.

The key difference between the predictions of inflation and string gas cosmology relates to the index of the spectrum. Whereas inflationary models always yield a red spectrum, string gas cosmology generically produces a blue shift (which makes it easier to detect on shorter wavelengths - see [51] for some current bounds). Based on this difference, a key challenge for experimentalists will be not only to detect the gravitational wave spectrum through B-mode polarization, but also to measure the spectral slope. If the results indicate a blue spectrum, the entire inflationary paradigm of structure formation will have been ruled out, and the experiments would have detected an effect which was first predicted from superstring theory [50].

### C. Gravitational Waves from a Matter Bounce

In the case of the matter bounce scenario, the evolution of the spectrum of gravitational waves and of cosmological perturbations is identical on super–Hubble scales during the contracting phase. If the bounce phase is short,

the evolution in this phase will not lead to a change in the amplitude of the fluctuations (neither of cosmological perturbations nor of gravitational waves). Thus, rather generically a matter bounce scenario will lead to a scale-invariant spectrum of gravitational waves with a large amplitude ( $r$ of the order 1). Taking into account the presence of entropy modes will lead to a reduction in the value of $r$ [52], but nevertheless the predicted amplitude of the gravitational wave spectrum is generically larger than what is predicted in inflationary models.

The spectrum of fluctuations produced by the matter bounce can be distinguished from that generated by simple single field inflationary models in terms of the induced non-Gaussianities. Specifically, the matter bounce predicts a special shape [53] of the bispectrum of cosmological fluctuations. The difference in shape arises in the following way: the general expression for the bispectrum is a function of the curvature fluctuation variable $\zeta$ and its time derivative $\dot{\zeta}$ [54]. In an expanding inflationary universe, $\zeta$ is constant on super-Hubble scales in the absence of entropy fluctuations, whereas in a contracting universe the dominant mode of $\zeta$ is increasing. Since the terms involving $\dot{\zeta}$ lead to a diferent shape of the bispectrum than that generated by the other terms, a matter bounce can in principle be distinguished from inflationary cosmology by measuring the shape of the bispectrum.

### D. No Gravitational Waves in the Ekpyrotic Scenario

In the case of the Ekpyrotic scenario, the equation of motion of the cosmological perturbations depends on the potential of the scalar field, whereas that of the gravitational waves does not.

If we start out with vacuum fluctuations for both cosmological fluctuation and gravitational wave modes on sub-Hubble scales in the contracting phase, then, unlike in the Matter Bounce scenario, the increase of the amplitude of the gravitational wave modes on super-Hubble scales is insufficient to turn the initial vacuum spectrum into a scale-invariant one. The predicted spectrum of gravitational waves is very blue and hence primordial gravitational waves are negligible on scales of cosmological interest today. In contrast, the cosmological perturbation modes couple to the potential and this allows them to attain a scale-invariant spectrum.

### VI. CONCLUSIONS

In this Note I have shown that there are many sources of gravitational waves which could lead to a roughly scale-invariant spectrum on cosmological scales. Thus, the detection of relic gravitational radiation via B-mode polarization will NOT prove inflation. Since several of the mechanisms described here predict a spectral amplitude which is larger than that generated in the simplest

inflationary models, one may argue that it is more likely that a positive signal will be due to a source different from inflation. Luckily, the different sources of gravitational waves - all of them giving a roughly scale-invariant spectrum - lead to specific predictions with which they can be distinguished. String gas cosmology produces a slight blue tilt of the spectrum, whereas inflationary cosmology always produces a slight red tilt. Cosmic strings lead to specific non-Gaussian signatures which can be identified in position space (see e.g. [44, 46] for studies of how to identify the cosmic string signal in CMB temperature maps). The matter bounce scenario leads to a particular shape of the bispectrum [53].

It must also be kept in mind that gravitational radiation is not the only way to generate primordial B-mode polarization. Once again, cosmic strings formed in a phase transition during the early Standard Cosmology phase of the evolution of the universe will generate primordial polarization which is statistically equally distributed between E-mode and B-mode [47]. This mechanism produces edges in CMB polarization maps - a signal which is easy to look for with position space based algorithms.

The message which this Note is supposed to convey is the following: the search for B-mode polarization is an extremely interesting field, much more interesting than if the only source of such primordial polarization were gravitational waves from inflation. It will be very important to carefully analyze the data without the prejudice that inflation is the only source of a signal. Otherwise, the possible existence of cosmic strings could be missed. More strikingly, if B-mode polarization is found and is shown to be due to gravitational waves, then if the spectrum is slightly blue one would have falsified the inflationary paradigm.

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[55] The word "approximately" is to be understood slightly differently in the two contexts above. All simple inflationary models lead to a slight deviation from scale-invariance - this explains the meaning of "approximate" in the first case. The simplese inflationary models produce a purely adiabatic spectrum of fluctuations, but a generic more complicated model will produce an isocurvature component to the fluctuation spectrum - this is the way in which the second use of "approximate" should be understood.

[56] The Hubble radius separates small scales where fluctuations typically oscillate from large scales where the microphysical oscillations are frozen out and the evolution of the perturbations are dominated by the gravitational dynamics of the background. See [8] for an in-depth treatment of the theory of cosmological perturbations, and [9] for an introductory overview.

[57] In numbers, more than $70H^{-1}$ (where $H$ is the Hubble expansion rate during the inflationary phase), while inflation needs to last at least $50H^{-1}$ - there is a slight dependence of these numbers on the energy scale at which inflation takes place - we have assumed the standard value.

[58] The decrease of $T$ for very small values of $R$ is a consequence of the T-duality symmetry of string theory. Once $R$ becomes smaller than the string scale, it becomes thermodynamically favorable for the energy of the string gas to excite string modes which wind the box, since winding modes become light as $R$ decreases.

[59] As in the case of string gas cosmology, this scenario is free from the trans-Planckian problem for fluctuations provided that the energy scale of the bounce is lower than the Planck scale.

[60] Taking the numbers from the more updated simulations

of [42] might change the overall coefficient by an order of magnitude - reliable statements are impossible to make since there is still a lot of uncertainty about the details of the cosmic string loop distribution during the scaling regime.

[61] The reason for the blue tilt is easy to understand physically [50]: the amplitude of the gravitational waves is given by the anisotropic pressure fluctuations. Their amplitude, in turn, is proportional to the background pressure. The background pressure tends to zero as we go deeper back into the emergent phase. Hence, long wavelength fluctuations which exit the Hubble radius earlier (see Figure 4) obtain less gravitational wave power than short wavelength modes.